معهد العلوم الاقتصادية

السنة الأولى ماستر
إدارة أعمال

أدوات التوقع والاستشراف
الموسم الجامعي: 2022/2023

التصحيح النموذجي للامتحان

التمرين 1: (08 نقاط) البرهان على أن: $\hat{\beta} = \frac{Cov(xy)}{V(x)} = r_{xy}\frac{S_y}{S_x}$

1/ لدينا: (04 نقاط)

$$r_{xy} = \sqrt{\hat{\beta}\cdot\hat{\beta}'} = \sqrt{\frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(x_i-\bar{x})}}\sqrt{\frac{\Sigma(y_i-\bar{y})(x_i-\bar{x})}{\Sigma(y_i-\bar{y})^2}}$$ 0,25 0,25

$$\Rightarrow r_{xy} = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\Sigma(x_i-\bar{x})^2}\sqrt{\Sigma(y_i-\bar{y})^2}} \quad \ldots\ldots (1)$$ 0,25

- بقسمة العلاقة رقم ① على (n) نحصل على:

$$r_{xy} = \frac{Cov(x,y)}{\sqrt{V(x)}\sqrt{V(y)}} = \frac{Cov(x,y)}{S_x S_y}$$ 0,25

- 0,25 أما عند ضرب العلاقة رقم ① في العبارة $\left(\sqrt{\Sigma(x_i-\bar{x})^2}\right)^2$ تصبح:

$$r_{xy} = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})\cdot\sqrt{\Sigma(x_i-\bar{x})^2}}{\sqrt{\Sigma(x_i-\bar{x})^2}\sqrt{\Sigma(x_i-\bar{x})^2}\sqrt{\Sigma(y_i-\bar{y})^2}} \Rightarrow$$ 0,25

$$r_{xy} = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})\cdot\sqrt{\Sigma(x_i-\bar{x})^2}}{\Sigma(x_i-\bar{x})^2\sqrt{\Sigma(y_i-\bar{y})^2}}$$ 0,25

ونعلم أن : (2) — $\hat{\beta} = \frac{\sum (x_i-\bar{x})(y_i-\bar{y})}{\sum (x_i-\bar{x})^2}$ (0,25)

تصبح :

$$r_{xy} = \hat{\beta} \cdot \frac{\sqrt{\sum (x_i-\bar{x})^2}}{\sqrt{\sum (y_i-\bar{y})^2}} \quad \text{— (3)} \quad (0,25)$$

بقسمة البسط والمقام في العبارة رقم (3) على n تصبح : (0,25)

$$r_{xy} = \hat{\beta} \cdot \frac{\sqrt{\frac{\sum (x_i-\bar{x})^2}{n}}}{\sqrt{\frac{\sum (y_i-\bar{y})^2}{n}}} \Rightarrow r_{xy} = \hat{\beta} \cdot \frac{\hat{S}_x}{\hat{S}_y} \Rightarrow \boxed{\hat{\beta} = r_{xy} \cdot \frac{\hat{S}_y}{\hat{S}_x}}$$

(0,25) (0,25) (0,25)

أما عند قسمة العلاقة رقم (2) على n تصبح :

$$\hat{\beta} = \frac{\frac{\sum (x_i-\bar{x})(y_i-\bar{y})}{n}}{\frac{\sum (x_i-\bar{x})^2}{n}} \Rightarrow \boxed{\hat{\beta} = \frac{cov(x,y)}{V(x)}} \quad (0,25)$$

(0,25)

ومنه فإن :

$$\boxed{\hat{\beta} = \frac{cov(x,y)}{V(x)} = r_{xy} \cdot \frac{S_y}{S_x}} \quad (0,25)$$

وهو المطلوب إثباته

/ لدينا : (1 ن 04)

أي :

$$y_i = \alpha + \beta x_i + \varepsilon_i$$
$$y_1 = \alpha + \beta x_1 + \varepsilon_1$$
$$y_2 = \alpha + \beta x_2 + \varepsilon_2$$
$$\vdots$$
$$y_n = \alpha + \beta x_n + \varepsilon_n$$

$$\sum y_i = n\alpha + \beta \sum x_i + \sum \varepsilon_i \quad (0,25)$$

بالقسمة على n :

$$\sum y_i / n = \frac{n}{n}\alpha + \beta \sum x_i / n + \frac{\sum \varepsilon_i}{n}$$

$$\Rightarrow \boxed{\bar{y} = \alpha + \beta \bar{x} + \bar{\varepsilon}} \quad \text{— (1)} \quad (0,25)$$

(0,25)

$$(y_i - \bar{y}) = (\alpha + \beta x_i + \varepsilon_i) - (\alpha + \beta \bar{x} + \bar{\varepsilon}) = (\alpha - \alpha) + \beta(x_i - \bar{x}) + (\varepsilon_i - \bar{\varepsilon})$$

$$\Rightarrow \boxed{(y_i - \bar{y}) = \beta(x_i - \bar{x}) + (\varepsilon_i - \bar{\varepsilon})} \quad (0,25)$$

نعلم سابقاً أن:

$\hat{\beta} = \sum w_i y_i$ 0,25

حيث:

$w_i = \frac{(x_i - \bar{x})}{\sum (x_i - \bar{x})^2}$ 0,25

$y_i = (Y_i - \bar{Y})$ 0,25

$\Rightarrow \hat{\beta} = \sum w_i \left[\beta (x_i - \bar{x}) + (\varepsilon_i - \bar{\varepsilon}) \right]$ 0,25

$\Rightarrow \boxed{\hat{\beta} = \beta \sum w_i (x_i - \bar{x}) + \sum w_i (\varepsilon_i - \bar{\varepsilon})}$ 0,25

نعلم أيضا أن:

$\sum w_i = 0$

$\sum w_i (x_i - \bar{x}) = 1$ 0,25

يعني أن:

$\hat{\beta} = \beta + \sum w_i (\varepsilon_i - \bar{\varepsilon}) = \beta + \sum w_i \varepsilon_i + \sum w_i \bar{\varepsilon}$ 0,25

$\Rightarrow \hat{\beta} = \beta + \sum w_i \varepsilon_i + \bar{\varepsilon} \sum w_i \Rightarrow \boxed{\hat{\beta} = \sum w_i \varepsilon_i}$ 0,25

0,25

لمعرفة تحيز $(\hat{\beta})$ يجب حساب توقعه الرياضي أي: $E(\hat{\beta}) = ?$

حيث:

0,25 $E(\hat{\beta}) = E(\beta + \sum w_i \varepsilon_i) = E(\beta) + E(\sum w_i \varepsilon_i)$

$\Rightarrow E(\hat{\beta}) = \beta + \sum w_i E(\varepsilon_i) \Rightarrow \boxed{E(\hat{\beta}) = \beta}$ 0,25

وهذا يعني أن $(\hat{\beta})$ هو مقدر غير متحيز لـ β 0,25

التمرين II: (12 نقطة)

1/ يعطى تباين $(\hat{\beta})$ وفق العلاقة التالية:

$\hat{\sigma}^2_{\hat{\beta}} = \frac{\hat{\sigma}^2_{\varepsilon}}{\sum (x_{1t} - \bar{x}_1)^2}$ 0,25

حيث: $\boxed{\hat{Y} = 10X_1 - 6}$, $R^2 = r^2_{Y,X_1} = 0,75$, $n = 100$, $\hat{\beta} = 10$

لدينا:

0,25

$* V(Y)$ $\frac{\sum (Y_t - \bar{Y})^2}{n} = 1000 \Rightarrow 1000 = \frac{\sum (Y_t - \bar{Y})^2}{100}$

$\Rightarrow \boxed{\sum (Y_t - \bar{Y})^2 = 100{,}000}$ 0,25

$* r^2_{Y,X_1} = R^2 = \hat{\beta}^2 \cdot \frac{\sum (X_{1t} - \bar{X}_1)^2}{\sum (Y_t - \bar{Y})^2} \Leftrightarrow 0,75 = (10)^2 \frac{\sum (X_{1t} - \bar{X}_1)^2}{100{,}000}$ 0,25

$\Rightarrow \sum (X_{1t} - \bar{X}_1) = \frac{75000}{100} \Rightarrow \boxed{\sum (X_{1t} - \bar{X}_1)^2 = 750}$ 0,25

$$* \ \hat{\beta} = \frac{\Sigma(x_{1t}-\bar{x}_1)(y_t-\bar{y})}{\Sigma(x_{1t}-\bar{x}_1)^2} \ (0,25) \iff 10 = \frac{\Sigma(x_{1t}-\bar{x}_1)(y_t-\bar{y})}{750} \ (0,25)$$

$$\Rightarrow \boxed{\Sigma(x_{1t}-\bar{x}_1)(y_t-\bar{y}) = 7500} \ (0,25)$$

$$* \ Cov(x_1, y) = \frac{\Sigma(x_{1t}-\bar{x}_1)(y_t-\bar{y})}{n} \ (0,25) = \frac{7500}{100} \Rightarrow \boxed{Cov(y, x_1) = 75} \ (0,25)$$

$$* \ V(x_1) = \frac{\Sigma(x_{1t}-\bar{x}_1)^2}{n} \ (0,25) = \frac{750}{100} \Rightarrow \boxed{V(x_1) = 7,5} \ (0,25)$$

$$* \ R^2 = r_{y.x_1} = 0.75 = 1 - \frac{\Sigma e_t^2}{\Sigma(y_t-\bar{y})^2} \ (0,25) \Rightarrow (1-0,75) = \frac{\Sigma e_t^2}{\Sigma(y_t-\bar{y})^2}$$

(0,25)

$\Leftarrow \Sigma(y_t-\bar{y})^2 = 100\,000$ (0,25) لأنّ معطى

$$\Sigma e_t^2 = (1-0,75)\,100,000 \Rightarrow \boxed{\Sigma e_t^2 = 25\,000} \ (0,25)$$

$$\hat{\delta}^2_{\varepsilon} = \frac{\Sigma e_t^2}{n-2} \ (0,25) \Rightarrow \hat{\delta}^2_{\varepsilon} = \frac{25000}{100-2} = \frac{25000}{98} = 255,1 \ (0,25)$$ لدينا

$$\Rightarrow \boxed{\hat{\delta}^2_{\varepsilon} = 255,1} \ (0,25) \Rightarrow \hat{\delta}^2_{\hat{\beta}} = \frac{\hat{\delta}^2_{\varepsilon}}{\Sigma(x_{1t}-\bar{x}_1)^2} = \frac{255,1}{750} = 0,34 \Rightarrow \boxed{\hat{\delta}^2_{\hat{\beta}} = 0,34} \ (0,25)$$

$$* \ t_{cal} = \left| \frac{\hat{\beta}}{\hat{\delta}_{\hat{\beta}}} \right| \ (0,25) = 10 / \sqrt{0,34} = 10 / 0,58 = 17.14$$

$$\Rightarrow \boxed{t_{cal} = 17,14} \ (0,25)$$

$$* \ t_{tab} = t_{98,\ 0,05} = 1,96 \Rightarrow \ (0,25)$$

$$t_{cal} = 17,14 > t_{tab} = 1,96 \ (0,25)$$

وهذا يعني أن المعلمة $(\beta=10)$ لها معنوية إحصائية عند مستوى 5% وهي تختلف عن الصفر.

(0,25)

2/ لدينا : (04 نقاط)

$$\hat{Y} = \hat{\alpha} + 4X_2$$

حيث : $\hat{\beta} = 4$, $\bar{X}_2 = 1$, $\bar{Y} = 12$ و

* 0,25

$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}_2 \Rightarrow \hat{\alpha} = 12 - 4(1) = 8 \Rightarrow \boxed{\hat{\alpha} = 8}$$ 0,25

$$\Rightarrow \boxed{\hat{Y} = 8 + 4X_2}$$ 0,25

* $$r^2_{Y,X_2} = 0,85 = \hat{\beta}^2 \cdot \frac{\sum(X_{2t} - \bar{X})^2}{\sum(Y_t - \bar{Y})^2} = (4)^2 \frac{\sum(X_{2t} - \bar{X})^2}{100,000}$$ 0,25

$$\Rightarrow \sum(X_{2t} - \bar{X}_2)^2 = 100,000 \times 0,85 / 16 \Rightarrow \boxed{\sum(X_{2t} - \bar{X}_2)^2 = 5312,5}$$ 0,25 0,25

* $$\hat{\beta}^2 = \frac{\sum(X_{2t} - \bar{X}_2)(Y_t - \bar{Y})}{\sum(X_{2t} - \bar{X})^2} \Leftrightarrow 4 = \frac{\sum(X_{2t} - \bar{X}_2)(Y_t - \bar{Y})}{5312,5}$$ 0,25

$$\Rightarrow \boxed{\sum(X_{2t} - \bar{X}_2)(Y_t - \bar{Y}) = 21250}$$ 0,25

* $$Cov(X_2, Y) = \sum(X_{2t} - \bar{X}_2)(Y_t - \bar{Y}) / n = \frac{21250}{100} = 212,5$$ 0,25

* $$V(X_2) = \frac{\sum(X_{2t} - \bar{X}_2)}{n} = \frac{53125}{100} = 53,125$$ 0,25

* $$\sum e_t^2 = (1 - 0,85) \cdot 100,000 \Rightarrow \boxed{\sum e_t^2 = 15000}$$ 0,25

* $$\hat{S}^2_{\hat{\beta}} = \frac{\hat{S}^2_{e_t}}{\sum(X_{2t} - \bar{X}_2)^2} \Rightarrow \hat{S}^2_{e_t} = \frac{\sum e_t^2}{n-2} = \frac{15000}{98} = 153,06$$ 0,25

$$\Rightarrow \hat{S}^2_{\hat{\beta}} = \frac{153.06}{5312,5} \Rightarrow \boxed{\hat{S}^2_{\hat{\beta}} = 0,03}$$ 0,25

* $$t_{cal} = \left| \frac{\hat{\beta}}{\hat{S}_{\hat{\beta}}} \right| = \frac{4}{\sqrt{0,03}} = \frac{4}{0,17} = 23,05$$ 0,25

* $$t_{tab} = t_{98, 0,05} = 1,96$$

$$t_{cal} = 23,05 > t_{tab} = 1,96$$ 0,25

وهذا يعني أن $(\hat{\beta} = 4)$ لها معنوية إحصائية عند مستوى 5%

وهي تختلف عن الصفر . 0,25

3/ حساب المعاملات $(\hat{\beta}_1, \hat{\beta}_2)$ للنموذج التالي: 02 نقطتين

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$$

لدينا:

* $\hat{\beta} = (X'X)^{-1} X'Y$ 0,25

من هذه العلاقة نحصل على:

$$\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} = \begin{bmatrix} Var(X_1) & Cov(X_1, X_2) \\ Cov(X_1, X_2) & Var(X_2) \end{bmatrix}^{-1} \begin{bmatrix} Cov(Y, X_1) \\ Cov(Y, X_2) \end{bmatrix}$$ 0,25

حيث: $Var(X_1) = 7,5$, $Var(X_2) = 53.12$, $Cov(X_1, X_2) = ?$

$Cov(Y, X_1) = 75$, $Cov(Y, X_2) = 212,5$

* $R^2 = r^2_{X_1, X_2} = \left[\dfrac{Cov(X_1, X_2)}{\hat{S}_{X_1} \hat{S}_{X_2}}\right]^2 = \dfrac{[Cov(X_1, X_2)]^2}{V(X_1) \cdot V(X_2)} = 0,45$ 0,25 0,25

$$\Rightarrow r^2_{X_1, X_2} \; \frac{[Cov(X_1, X_2)]^2}{7,5 \cdot 53,12} = 0,45$$

$$\Rightarrow \left[\sum (X_{1t} - \bar{X}_1)(X_{2t} - \bar{X}_2)/n\right]^2 = 7,5 \cdot 53,12 \cdot 0,45 = 179,28$$ 0,25

$$\Rightarrow Cov(X_1, X_2) = \sqrt{179,28} \Rightarrow Cov(X_1, X_2) = 13,389$$ 0,25

$$\Rightarrow \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} = \begin{bmatrix} 7,5 & 13,39 \\ 13,39 & 53,12 \end{bmatrix}^{-1} \begin{bmatrix} 75 \\ 212,5 \end{bmatrix} = \begin{bmatrix} 0,242 & -0,061 \\ -0,061 & 0,034 \end{bmatrix} \begin{bmatrix} 75 \\ 212,5 \end{bmatrix}$$ 0,25

$$\Rightarrow \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} = \begin{bmatrix} 5,19 \\ 2,69 \end{bmatrix} \Rightarrow \hat{\beta}_1 = 5,19 \; ; \; \hat{\beta}_2 = 2,69$$ 0,25